Islamic University of Gaza
Statistics, ECIV 2305
First Semester (2007-2008)
Midterm Exam
Allowable Time: 75[min]

Instructor:
Dr. Yunes Mogheir

CLOSED BOOK EXAM

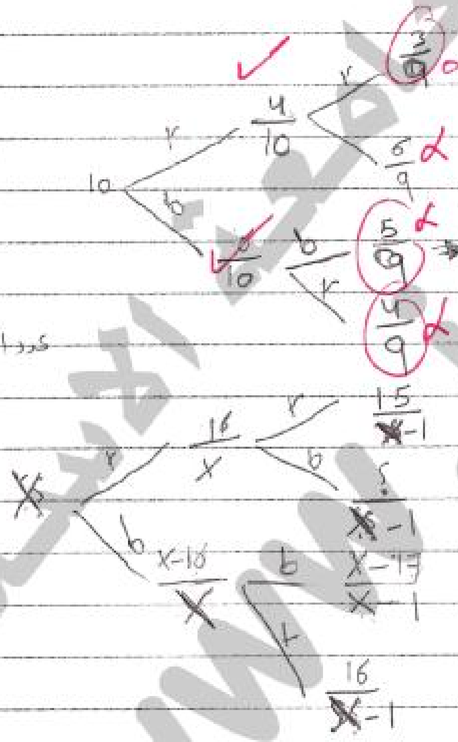
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Student Name: Student No.:

Question 1 **(8 Marks)** 4

A box contains 10 balls: 4 red and 6 blue. A second box contains 16 red balls and an unknown number of blue balls. A single ball is drawn from each box. The probability that both balls are the same color is 0.44. **Calculate the number of blue balls in the second box.**

box: 10 balls — 4 red, 6 blue
box: 16 red, ?? blue



$\frac{5}{9} \times \frac{6}{10} + \frac{3}{9} \times \frac{4}{10} = 0.46$

$\frac{15}{X-1} \times \frac{16}{X} + \frac{X-15}{X-1} * \frac{X-16}{X}$

-4

$0.46 + \frac{15}{X-1} * \frac{16}{X} + \frac{X-15}{X-1} * \frac{X-16}{X} = 0.44$

6

Question 2

(6 Marks)

You are given $P[A \cup B] = 0.7$ and $P[A \cup B'] = 0.9$.
Determine P[A]?

$p(A \cup B) = p(A) + p(B) - p(A \cap B)$
$p(A \cup B') = p(B') + p(A \cap B)$ ✓

$p(B) = p(A \cup B') - p(A \cap B)$ ✓

$p(A' \cup B) = p(A') + p(A \cap B)$

$p(A) = p(A \cup B') - (1 - p(A \cup B))$
$= 0.9 - 0.3$
$= 0.6$ ✓

Question 3

$12\frac{1}{2}$

(15 Marks)

Suppose the cumulative distribution function of the random variable X is:

$$F(X) = \begin{cases} 0 & x < -2 \\ A\,x + 0.5 & -2 \le x < 2 \\ B & 2 \le x \end{cases}$$

1) Find the value of A and the expected value of B? 2

$F(2) \Rightarrow Ax + 0.5 = 1$ $B = 1$
$2A + 0.5 = 1$
$A = 0.25$
−1

$$F(x) = \begin{cases} 0 & x < -2 \\ 0.25x + 0.5 & -2 \le x < 2 \\ 1 & 2 \le x \end{cases}$$

2) What is the probability $P(-1 \leq x \leq 1)$? (0.5)

$$\int_{-1}^{1} (0.25x + 0.5)\,dx \Rightarrow \frac{0.25x^2}{2} + 0.5x \Big|_{-1}^{1}$$

$$\frac{0.25}{2} + 0.5 - \left(\frac{0.25}{2} - 0.5\right) \Rightarrow \frac{0.25}{2} + 0.5 - \frac{0.25}{2} +$$

$= \boxed{1}$ ✗

3) Calculate the variance of this random variable? (3)

$Var(x) = E(x^2) - (E(x))^2$

$f(x) = 0 \quad 2 \leq x$

$Ax + 0.5 \quad -2 \leq x \leq 2$

$f(x) = \frac{dF(x)}{dx} \Rightarrow 0.25 \quad f(x) = 0.25$

$$E(x) \int_{-2}^{2} 0.25\,(x)\,dx \Rightarrow \frac{0.25x^2}{2}\Big|_{-2}^{2} \Rightarrow \frac{1}{2} - \frac{1}{2} = 0$$

$$E(x^2) \int_{-2}^{2} 0.25x^2\,dx \Rightarrow \frac{0.25x^3}{3}\Big|_{-2}^{2} \Rightarrow \frac{2}{3} - \left(\frac{-2}{3}\right) = 1.333$$

$Var(x) = 1.333 - 0^2 = 1.333$

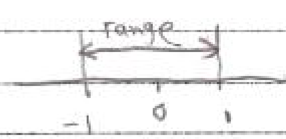
4) What is the interquartile range? (3)

$0.25x + 0.5 = 0.25$

$x = -1$

$0.25x + 0.5 = 0.75$

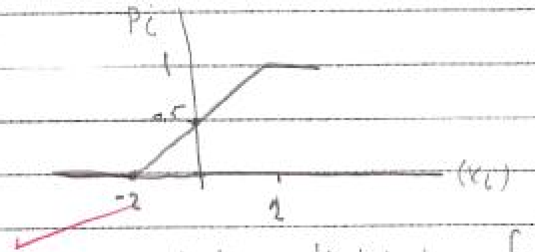
$x = 1$



rang = 2

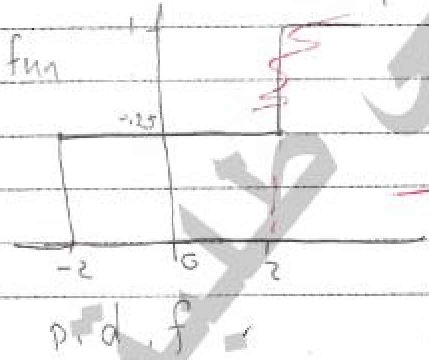
5) Sketch the probability density function and the cumulative distribution function?

2 1/2



Cumulative distribution fun

(x)	-2	0	2
P_i	0.25	0.25	0.25
$F(x_i)$	0	0.5	1



−1/2

p.d.f ✓

Question 4 **(6 Marks)** 3½

In Islamic University Library, 6 statistic books and 6 structure books are available; **in how many ways 4 books** from the two types of books can be selected if

a) The selected books must consist of two statistics and two structures books?

$C^6_4 = \frac{6!}{4!\,2!} = 15$ $\quad P^6_4 = \frac{6!}{4!} = 30$

$15 \times 2 = 30$

$C^4_2 = \frac{4!}{2!\,2!} = 6$ $\quad 6 \times 2 = 12$ $\quad P^4_2 = \frac{4!}{2!} = 12$

$P^6_2 = \frac{6!}{2!} = 360$

1

b) There must be at least one each type of books?

$C^4_1 = \frac{4!}{1!\,3!} = 4$

$\binom{6}{3} \times \binom{6}{1} + \binom{6}{2} \times \binom{6}{2} + \binom{6}{1}\binom{6}{3}$

1½